Repeated-measures ANOVA

- Data for each new experimental condition is provided by testing a completely new and independent set of subjects
- "Non-repeated measures" or "Between subjects" design
- **Independent** or **Non-repeated Measures ANOVA**

- When subjects are tested on two or more occasions
- "Repeated measures" or "Within subjects" design
- **Repeated Measures ANOVA**
- But, … we have a problem…
- **"Sphericity"**

Rough Definition...

	Delay		
	D1	D2	D3
S1	10	9	8
S2	10	8	10
S3	9	9	8
S4	10	5	1
S5	9	6	0
S6	8	5	0
S7	9	1	0
S8	10	0	1
S9	10	2	0

	Differences		
	D1-D2	D1-D3	D2-D3
S1	1	2	1
S2	2	0	-2
S3	0	1	1
S4	5	9	4
S5	3	9	6
S6	3	8	5
S7	8	9	1
S8	10	9	-1
S9	8	10	2

In other words, we assume that the effect of the manipulation (in this case 'delay') is the approximately same for all participants

For Sphericity, standard deviations of these three columns must be equal

Assessing Sphericity

- One approach is to use “Mauchly’s test of Sphericity”
- A significant Mauchly W indicates that the sphericity assumption has been violated
- “Significant W = trouble”
- **But…** Mauchly’s test is inaccurate
- What do we do instead?
- Evaluate “sphericity” (“lower bound”, Greenhouse-Geisser, Huyn-Feldt)

Dealing with departures from Sphericity assumptions

- Worst Case Scenario:
  - *This assumes that the violation of sphericity is as bad as it could possibly be*
- In other words, each participant is affected entirely differently by the manipulation
- This is known as the "Lower Bound"
- For ANOVA procedures with repeated-measures, four different *F*-ratios and *p-values* can be reported

Dealing with departures from Sphericity assumptions

Tests of Within-Subjects Effects

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig
A	Sphericity Assumed	59.583	3	19.861	19.324	.002
	Greenhouse-Geisser	59.583	1.213	49.138	19.324	.033
	Huynh-Feldt	59.583	2.080	28.648	19.324	.008
	Lower-bound	59.583	1.000	59.583	19.324	.048
Error(A)	Sphericity Assumed	6.167	6	1.028		
	Greenhouse-Geisser	6.167	2.425	2.543		
	Huynh-Feldt	6.167	4.160	1.482		
	Lower-bound	6.167	2.000	3.083		

Result is highly significant if it is safe to assume there is no Sphericity violation

But only just significant if we assume the worst possible violation of the assumption

G-G and H-F significance levels are intermediate

Dealing with departures from Sphericity assumptions

Tests of Within-Subjects Effects

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
A	Sphericity Assumed	59.583	3	19.861	19.324	.002
	Greenhouse-Geisser	59.583	1.213	49.138	19.324	.033
	Huynh-Feldt	59.583	2.080	28.648	19.324	.008
	Lower-bound	59.583	1.000	59.583	19.324	.048
Error(A)	Sphericity Assumed	6.167	6	1.028		
	Greenhouse-Geisser	6.167	2.425	2.543		
	Huynh-Feldt	6.167	4.160	1.482		
	Lower-bound	6.167	2.000	3.083		

Notice that the only difference between the four tests lies in the degrees of freedom

..and there is no difference in the F-ratios themselves

Alternative: Assume the Worst! (Total Lack of Sphericity)

- Conservative F Test (1958)
  - ➔ *Provides a means for calculating a correct critical F value under the assumption of a complete lack of sphericity (lower bound):*

$F_{crit}(1, df_A)$

where

$df_A = a - 1 =$ number of subjects - 1

Estimating Sphericity

- What if your F statistic falls between the 2 critical values (assuming sphericity or assuming total lack of sphericity)?

- Solution: estimate sphericity, and use estimate to adjust critical value.

$$\text{Sphericity parameter } \varepsilon : \frac{1}{df_B} \le \varepsilon \le 1$$

$$F_{crit}(df_B, df_A \times df_B) \rightarrow F_{crit}(\varepsilon \times df_B, \varepsilon \times df_A \times df_B)$$

- Two different methods for calculating ε:
  - Greenhouse and Geisser (1959)
  - Huynh and Feldt (1976) – less conservative

Example: Grating Detection

			Noise			Group Total
			.04	.15	.50	
						Mean
Subject	1		.08480	.06830	.06540	.07283
	2		.08290	.06090	.07610	.07330
	3		.08880	.06440	.07120	.07480
Group Total		Mean	.08550	.06453	.07090	.07364

Mauchly's Test of Sphericity

Measure: MEASURE_1

Within Subjects Effect	Mauchly's W	Approx. Chi-Square	df	Sig.	Epsilon[a]		
					Greenhouse-Geisser	Huynh-Feldt	Lower-bound
noise	.383	.960	2	.619	.618	1.000	.500

Tests the null hypothesis that the error covariance matrix of the orthonormalized transformed dependent variables is proportional to an identity matrix.

a. May be used to adjust the degrees of freedom for the averaged tests of significance. Corrected tests are displayed in the Tests of Within-Subjects Effects table.

Tests of Within-Subjects Effects

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
noise	Sphericity Assumed	.001	2	.000	14.355	.015
	Greenhouse-Geisser	.001	1.237	.001	14.355	.044
	Huynh-Feldt	.001	2.000	.000	14.355	.015
	Lower-bound	.001	1.000	.001	14.355	.063
Error(noise)	Sphericity Assumed	9.66E-005	4	2.41E-005		
	Greenhouse-Geisser	9.66E-005	2.473	3.91E-005		
	Huynh-Feldt	9.66E-005	4.000	2.41E-005		
	Lower-bound	9.66E-005	2.000	4.83E-005		

SS_B

SS_{resid}

Impiegando R (e rmf)

```
> y <- c(8480,6830,6540,8290,6090,7610,8880,6440,7120)
> (Y <-matrix(y,ncol=3,byrow=TRUE)/10000)
      [,1]  [,2]  [,3]
[1,] 0.848 0.683 0.654
[2,] 0.829 0.609 0.761
[3,] 0.888 0.644 0.712
>
> # GG e HF
> aov.mr(Y)
Analysis of Variance Table

Greenhouse-Geisser epsilon: 0.6183
Huynh-Feldt epsilon:        1.1202

  Res.Df Df  Gen.var.      F num Df den Df   Pr(>F)   G-G Pr   H-F Pr
1      2    0.0008626
2      3  1 0.0047325 14.355      2      4 0.014954 0.044428 0.014954
>
> # lower bound
> eps <- 1/(3-1)
> pf(14.355,2*eps,4*eps,lower.tail=FALSE)
[1] 0.06313649
```